Jet-Driven Resonance Tube

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Some experimental results for the resonance tube are discussed. Strong periodic oscillations with internal shock waves are observed, together with temporarily steady external flow phases. By using simple boundary conditions, the internal flow may be calculated, such calculation showing good agreement with experiment. The trapped indigenous fluid is maximally compressed to 30% of the tube length.

Introduction

EXAMPLES of the oscillation of chambered pneumatic devices such as the Helmholtz resonator are numerous, even outside the field of music. Most such devices are acoustic or linear. This paper discusses what might be called a "nonlinear organ pipe" or resonance tube.

A resonance tube is simply a cylindrical cavity, closed at one end. If a supersonic flow is directed axially against the open end, the gases within the tube are, under a broad range of resonant conditions, excited to violent motion (Fig. 1). The term resonance tube (Resonanzrohr) was applied by Sprenger,[1] who extensively described the case of a tube excited by the structured flow from an underexpanded nozzle. A striking demonstration mentioned by Sprenger consists in using a blind hole in a piece of wood as the resonance tube; after the jet has operated for a short time, the wood bursts into flames! Such heating effects are principally associated with internal shock waves. A careful experimental investigation of energy dissipation within a resonance tube has been made by Sibulkin.[2]

The resonant flow in a short tube was first described by Hartmann,[3] who also used the underexpanded nozzle. Tubes excited by a supersonic freestream have been described by Vrebalovich.[4, 5]

Resonant oscillation within a piston-drive tube has also been considered.[6–9] In this case also, internal shock waves of moderate strength may appear. Particle displacement amplitudes are small, however, in comparison with the resonance tube. Furthermore, the resonance tube has three-dimensional unsteady flow at the open end, as opposed to the piston-driven tube, so that the boundary conditions are different.†

Most of the effects associated with underexpanded-jet excitation of a resonance tube can be reproduced on the two-dimensional water table.[10]

This paper considers the motion induced in a circular tube excited by a parallel supersonic jet of the same diameter. The spectacular features of the flow are the great noise produced (~120 db or more) and the considerable temperature rise[11] of the tube walls near the closed end ($\Delta T \sim 400$°F or more). Any satisfactory "theory" must, at least, account for these effects.

General Features of the Flow

The essentially one-dimensional motion of gas within the tube is described by well-known characteristic differential equations[12]; thus, the internal flow can be reconstructed provided that suitable characterization of the heat transfer is possible *and* that the complicated external flow behavior near the tube mouth can be described by some boundary conditions.

The experiments described here were made with a ½-in.-diam stainless-steel tube, 20 in. long (Fig. 1). The driving jet, at 1 atm, had Mach number 1.92. Shadowgraphs of the external flow (Fig. 2) show that the inflow and outflow phases are temporarily steady. Transition to outflow is announced by discharge of a moderately strong shock, this being the main internal shock. The transition to inflow is much more gradual, and advantage of this will be taken by treating this phase as quasi-steady.

Internal Flow

The questions of open-end boundary conditions and heat transfer will be discussed in turn.

Consider the outflow phase. As long as the tube discharge is supersonic, the flow is autonomic, because there are two characteristics of opposite family coming from the interior of the tube. As the outflow weakens, however, the jet collision system must move toward the mouth of the tube and influence the tube discharge. Now the external flow in this phase may be considered quasi-steady,‡ so that discharge conditions conform to the situation in which the tube outflow has some fixed fraction of the jet strength. As long as the collision structure is maintained, that is, as long as outflow persists, there is a matching of jet impulse $J_{\text{tube}} \sim J_{\text{nozzle}}$. Otherwise, outflow cannot take place.

When the quasi-steady outflow velocity is just zero, stagnation conditions prevail at the tube mouth.§ The effective pressure on the tube face is, however, somewhat less than the jet stagnation pressure, because of streamline curvature. For the particular conditions of this report, pressure records taken near the open end indicate that the tube outflow impulse should be fixed at $J_{\text{tube}} = (P + \rho u^2) = 80\% \ P_{0j}$, where P_{0j} is the nozzle jet stagnation pressure.¶ The outflow boundary conditions ($M_e < 1$) are thereby fixed and cor-

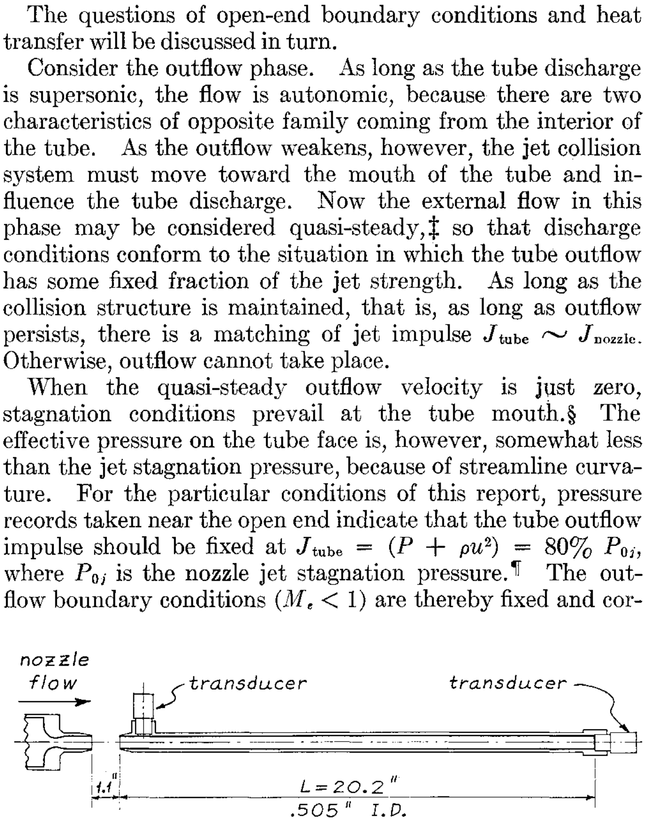

Fig. 1 Jet-driven resonance tube, experimental arrangement.

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† There is, of course, *some* piston motion that will give rise to an internal flow duplicating that within a resonance tube.

‡ Convective accelerations are of the order of 20 times the unsteady accelerations.

§ Frame 8 of Fig. 2 shows this stagnation condition. It is almost identical to a shadowgraph (not shown) of the steady stagnation flow against a closed resonance tube, that is, a tube of zero length.

¶ It is interesting to note that a constant impulse line coincides with a characteristic R^- at the sonic line (Fig. 3).

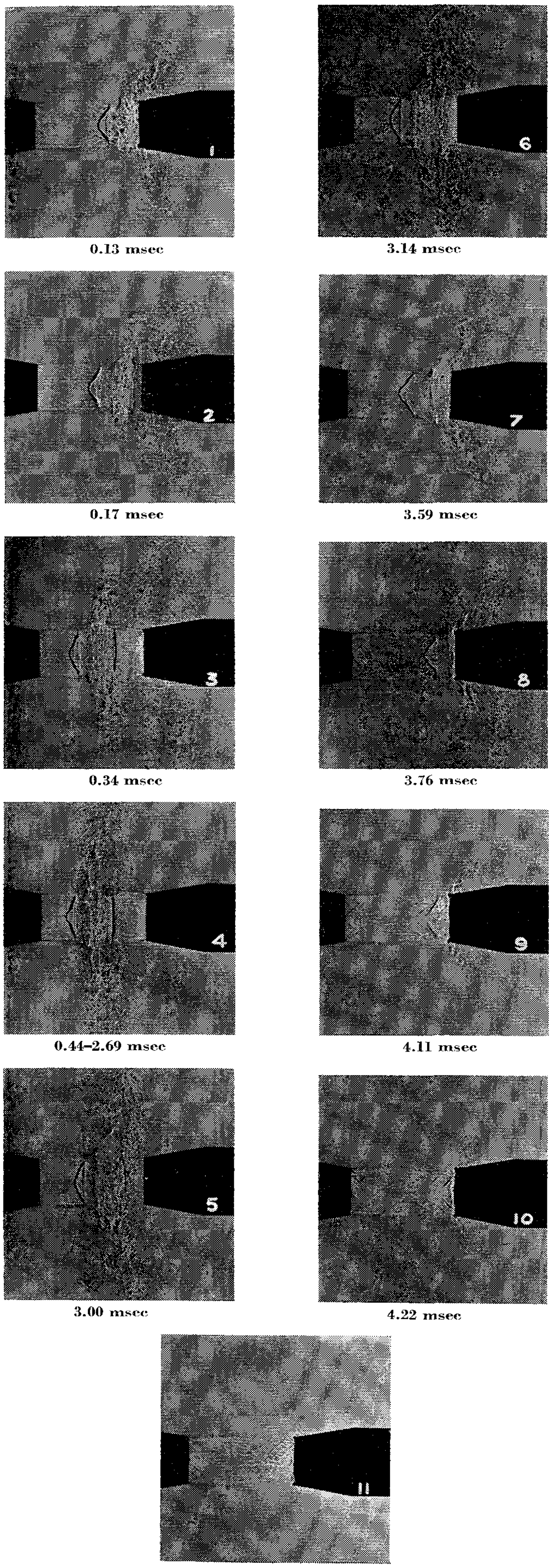

Fig. 2 Consecutive external flow shadowgraphs for one cycle (jet Mach number = 1.92).

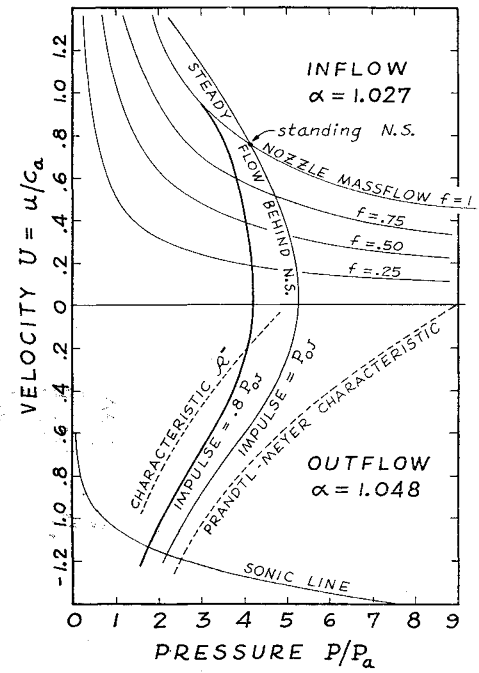

Fig. 3 Pressure-velocity boundary states for outflow-inflow transition.

respond to a generalization of the usual condition of constant open-end pressure.

The *inflow* should asymptotically approach the full jet mass flow and be continuous with the constant impulse outflow curve. A boundary-states curve meeting these requirements is sketched in Fig. 3. The curves shown on this figure relate velocity U to the thermodynamic variable P, with the entropy (as measured by α, defined below) held constant. The steady flow relations are $\rho u = (\rho u)_{\text{nozzle}} \times f$ (constant mass flow fractions) and $u^2/2 + C_pT = $ const (steady flow behind normal shock). The unsteady flow relations are $u - 2c/(\gamma - 1) = R^-$ (characteristic lines). Constant impulse lines and the sonic line correspond, respectively, to $P + \rho u^2 = $ const and $|u| = c$. The actual boundary states chosen for transition from outflow to inflow follow the heavy line, as discussed previously.

The internal flow may now be calculated if the fluid *entropy* is known. For the well-cooled tube used in these experiments, it is proposed to treat the internal flow as isentropic!

For periodic, steady-state operation there can be no accumulation of entropy within the tube; entropy generation due to shock waves** is just balanced by cooling of the fluid. A particular fluid particle has no net entropy change over one cycle; its entropy variation is of the order of the entropy change across a shock of pressure ratio 2.5, say. Such shocks may be treated as isentropic (in the sense that the Riemann invariant is unchanged) to a good approximation. Thus, although the major source of entropy generation is in the shockwave, *all* entropy generation is ignored for the purposes of calculating the internal flow dynamics.

Making use of the foregoing consideration, an internal wave diagram has been computed (Fig. 4). For isentropic flow of ideal gas, $P_\rho{}^{-\gamma} = $ const. This is written as

$$(P/P_a)^\gamma = \alpha^2(\rho/\rho_a) \tag{1}$$

where P_a is atmospheric pressure, ρ_a is the density in the correctly expanded supersonic nozzle jet, and α (of order unity) is a measure of the entropy rise of the fluid. Equations corresponding to (1) may be written for sound speed, tem-

** An order-of-magnitude calculation shows that boundary-layer heating accounts for only about 5–10% of the total.

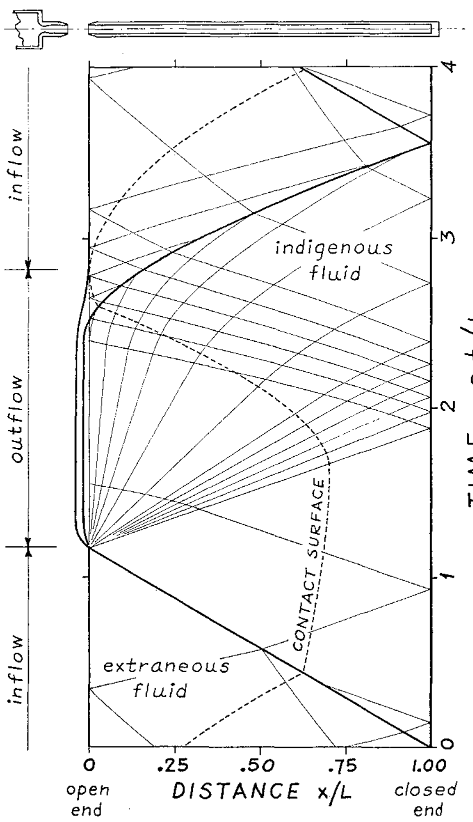

Fig. 4 Calculated wave diagram.

perature, etc. The characteristic inviscid equations of motion are

$$\left\{\frac{\partial}{\partial\tau} + (U \pm C)\frac{\partial}{\partial X}\right\}\left(U \pm \frac{2}{\gamma - 1}C\right) = 0 \qquad (2)$$

where U and C are normalized with respect to sound speed c_a in the jet, $\tau = c_a t/L$, and $X = x/L$.

The value of α for the internal flow calculations has been taken to be everywhere 1.048, corresponding to passage of the nozzle gas through a standing normal shock ($M = 1.92$)

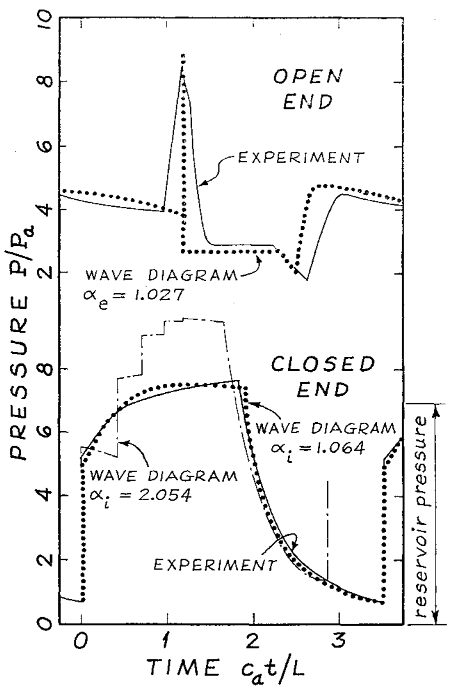

Fig. 5 Calculated and experimental pressure histories.

and a subsequent shock bringing the fluid to rest (pressure ratio 2.18). The internal fluid is either *indigenous* (permanently remaining in the tube) or *extraneous* (fluid from the nozzle remaining in the tube for only one cycle); the indigenous and extraneous fluid are separated by an entropy discontinuity or contact surface. Subsequent considerations will show empirically that under the experimental conditions α changes across this surface from about 1.027 to about 1.064. The conditions of pressure and velocity matching give that the change in a Riemann invariant across the contact surface is

$$\Delta\left(U \pm \frac{2}{\gamma - 1}C\right) = \frac{2}{\gamma - 1}C_1\left(1 - \frac{\alpha_2}{\alpha_1}\right) \qquad (3)$$

or about 1%. The invariant is thus, to a good approximation, truly invariant. Similar comments apply to a shock wave.

The wave diagram was computed from the following initial conditions: fluid from the nozzle passes through a normal shock at the tube mouth. This flow is brought to rest by a shock coming from the closed end. As this shock passes from the open end, the stagnant gas is accelerated by a centered Prandtl-Meyer expansion, which gives a sonic outflow. After three calculated cycles, the flow is already periodic, as shown in Fig. 4.

The *period* of the flow is of interest. The acoustic period is $4L/c_a$ or 7.5 msec. The experimental and calculated periods are both 6.8 msec.

The amplitude of fluid motion indicated in the wave diagram is surprisingly great. The contact surface is at $X \approx 0.70$ at maximum compression and, of course, at $X = 0$ at maximum expansion. The wave motion is distinctly nonlinear, as shown, for example, by the intersection of characteristic and shock lines.

Pressure History

The transducer records may be compared with calculations from the wave diagram. The parameter actually appearing in the computations is the dimensionless sound speed C, and this is related to the pressure by

$$P/P_a = (C/\alpha)^{2\gamma/(\gamma-1)} \qquad (4)$$

Even though α is of order unity, $2\gamma/(\gamma - 1)$ is seven (air); the pressure, therefore, is somewhat sensitive to α, and small differences must be taken into account. This has been done by setting $\alpha = 1.027$ (open end) and $\alpha = 1.064$ (closed end). These choices for α are empirical in that they give the best agreement between calculated and experimental pressure histories. The values chosen would depend, in general, on the driving jet, wall resistance to heat transfer, and so on.

The foregoing considerations apply only to a relatively well-cooled tube. High temperatures may be achieved by insulating the tube, as was done by Ackeret,[11] who used a tube constructed like a Thermos bottle. For this case, the preceding consideration may be applied by using a two-entropy model[13] in the wave-diagram calculations. Many shock reflections then occur at the contact surface, and the internal flow is more complicated. A pressure history computed from such a wave diagram is also shown in Fig. 5. Some

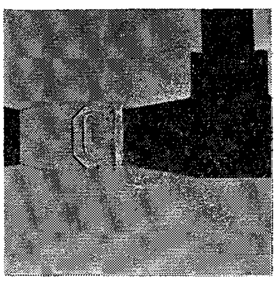

Fig. 6 Unstable shock pattern in periodic jet blunt-body flow (reservoir pressure 60 psig, sonic nozzle discharge).

approximate calculations for internal temperature distributions are given in Ref. 13.

Concluding Remarks

The calculation of internal motions may be successfully done by considering only nonlinear effects. This agrees with the findings of Betchov[6] and Chester,[9] who considered the effects of viscosity and heat conduction for a piston-driven resonator.

Although relatively high temperatures are indeed obtainable near the closed end,[1, 11] the "efficiency" of such a heating device is low. For the tube tested, the heating over one cycle $T_a \Delta S$ is of the order of 1% of kinetic energy discharge at the nozzle. Use is being made, however, of the resonance tube as a combustion igniter. Unfortunately, many of the "applications" seem to be in the form of unforeseen accidents[14] in pneumatic devices and in stability problems experienced with resonance-tube-like configurations.

The conditions under which a tube will resonate are of interest. This question has been investigated.[15] Some experiments were made by the author with a plugged tube, or "resonance tube of zero length." This blunt-body device gives an oscillatory flow when placed in the converging region of flow from an underexpanded nozzle (Fig. 6) but a stable flow when placed before a straight supersonic jet.

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